

(No calculators other than y-function)

Quiz 1, Linear - Fall 2017

11:31

11:38

7 min
⇒ 20/25 min
in
class.

Name: _____

Key

1. (4 points) Use matrices to solve the system. Show your work.

$$\begin{aligned}x_1 - 4x_3 &= 9 \\2x_1 + 2x_2 + 9x_3 &= 7 \\x_2 + 5x_3 &= -2\end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 9 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array} \right] \checkmark \checkmark$$

$$\begin{aligned} -2R_1 + R_2: \\ R_2 \leftrightarrow R_3: \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -4 & 9 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 17 & -11 \end{array} \right] \checkmark \checkmark \checkmark$$

$$-2R_2 + R_3 \rightarrow R_3 \left[\begin{array}{ccc|c} 1 & 0 & -4 & 9 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 7 & -7 \end{array} \right] \checkmark$$

$$\begin{aligned} \frac{1}{7}R_3 \rightarrow R_3 \\ \text{Back substitute} \end{aligned} : \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right] \checkmark \checkmark$$

$$x_1 = 5$$

$$x_2 = 3$$

$$x_3 = -1$$

2. (4 points) (a) Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.
 (b) Which value(s) of h , if any exist, correspond to a system with a unique solution?
 (c) Which value(s) correspond to a system with an infinite number of solutions?

$$\left[\begin{array}{cc|c} 1 & 4 & -2 \\ 2 & h & -4 \end{array} \right]$$

$$-2R_1 + R_2 \rightarrow R_2: \left[\begin{array}{cc|c} 1 & 4 & -2 \\ 0 & -8+h & 0 \end{array} \right] \checkmark$$

✓ (a) The system is always consistent b/c this is zero

✓ (b) if $h \neq 8$, then $-8+h$ is a pivot and you will simplify to $\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 0 \end{array} \right] \Rightarrow \begin{array}{l} x_1 = -2 \\ x_2 = 0 \end{array}$ is a unique solution

↓ (c) if $h = 8$, then get zero row on bottom $\Rightarrow x_2$ is free and have infinite # solutions

-0.5 if specific solution for (b)

-1 if wrong answer for (b).

-0.5 if no/little explanation for (a), (c),

3. (2 points) A system of linear equations with fewer equations (rows) than unknowns (columns) ^{but have correct answer} is called an *underdetermined* system. Suppose such a system happens to be consistent. Explain (briefly) why there must be an infinite number of solutions. Giving an example may help your explanation.

If the system is consistent, that means either there is a unique # of solutions or infinite. For a unique solution, must have a pivot in every column, but since there are more columns than rows, you must have some free variables \Rightarrow infinite # solutions.